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(May–June)

FYUP : 4th Semester Examination

PHYSICS

(Quantum Mechanics—I)

(PHY–252)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer *any* **ten** questions

1. *(a)* Discuss the failure of classical mechanics to explain black-body radiation and photoelectric effects. How was quantum mechanics able to explain these phenomena? 3+3=6

   (b) Photons of energy 1·02 MeV undergo Compton scattering through 180°. Calculate the energy of the scattered photon. 1½

2. (a) What are matter waves? Derive an expression for the de Broglie wavelength associated with an electron accelerated under a potential difference of V volts. 1+4=5

(b) Calculate the de Broglie wavelength of an α-particle accelerated through a potential difference of 4 kV. 2½

3. Define phase velocity and group velocity of a wave packet. Derive the expression for group velocity and hence its relation with phase velocity. 2+4+1½=7½

4. (a) Illustrate Heisenberg's uncertainty principle by single-slit electron diffraction experiment. 4½

(b) What do you understand by the wave function ψ of a moving particle? What are the conditions for the wave function to be well-behaved? 1+2=3

5. (a) State and explain the fundamental postulates of quantum mechanics. 5

(b) Normalize the wave function of a particle given by

$$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$$

where the particle is trapped in space between $x = 0$ and $x = L$, and A is a constant. 2½

6. State and prove Ehrenfest theorem. 7½

7. (a) Explain the term 'probability current density'. 1½

(b) Prove the relation

$$\frac{\partial P}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

where $\vec{J}$ is probability current density and P is the probability density. Giving one example, write the physical interpretation of the equation. 4+2=6

8. Write down the time independent Schrödinger equation in a one-dimensional system for a particle of mass m in an infinite potential well with

$$V(x) = 0, \quad 0 \le x \le a$$
$$V(x) = \infty, \quad \text{elsewhere}$$

and hence find the eigenvalues and normalized eigenfunctions. 7½

9. (a) What is a Hermitian operator? Show that the expectation value of a dynamic quantity represented by a Hermitian operator is always real. 1+3=4

(b) Show that the momentum operator $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ is Hermitian. 3½

10. Define expectation value of a dynamical variable. If

$$\psi(x) = \frac{\sqrt{a}}{\pi^{\frac{1}{4}}} e^{-a^2x^2/2}$$

find the values of $\langle x^2 \rangle$ and $\langle p_x \rangle$. 1½+3+3=7½

11. Prove Heisenberg uncertainty principle by operator method. 7½

12. Write down the expressions of the components of the orbital angular momentum in spherical polar coordinates and hence derive the expression of L^2. 7½

13. Show that $[\hat{L}^2, \hat{L}_z] = 0$ and give its physical significance. Also, determine the eigenvalues of $\hat{L}^2$ and $\hat{L}_z$. 2+1+2+2½=7½

14. *(a)* Obtain the following commutation relations for Pauli's spin matrices : 2×2=4

(i) $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\sigma_z$

(ii) $[\hat{\sigma}^2, \hat{\sigma}_z] = 0$

(b) If $\vec{A}$ and $\vec{B}$ are two vector operators which commute with $\vec{\sigma}$'s but not necessarily with each other, then show that

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = (\vec{A}\cdot\vec{B}) + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$ 3½

15. Write down the expressions of $\hat{J}_x$, $\hat{J}_y$ and $\hat{J}_z$, where the symbols have their usual meanings. Prove the following commutation relations : 1½+(2×3)=7½

(i) $[\hat{J}_z, \hat{J}_+] = \hbar\hat{J}_+$

(ii) $[\hat{J}^2, \hat{J}_+] = 0$

(iii) $[\hat{J}_+, \hat{J}_-] = 2\hbar\hat{J}_z$

16. *(a)* Using the Heisenberg's uncertainty principle, explain why free electrons cannot exist inside a nucleus. 2½

(b) Find the eigenvalues and normalized eigenvectors of the matrix

$$\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$ 5

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